LECTURE NO 7

Topics

- Del operator
- Advantages of Del operator
- Gradient
- Gradient of curl

Del Operator

The del operator, written ∇, is the vector differential operator. In Cartesian coordinates,

$$\nabla = \frac{\partial}{\partial x}\mathbf{a}_x + \frac{\partial}{\partial y}\mathbf{a}_y + \frac{\partial}{\partial z}\mathbf{a}_z$$

This vector differential operator, otherwise known as the *gradient operator*, is not a vector in itself, but when it operates on a scalar function, for example, a vector ensues. The operator is useful in defining

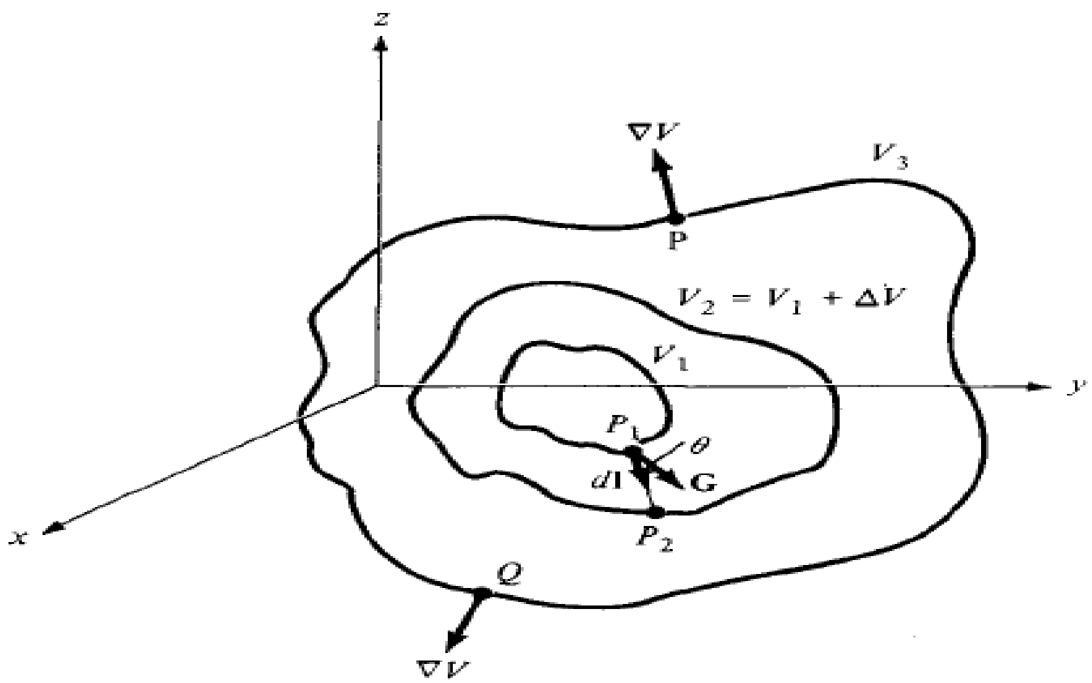
1. The gradient of a scalar V, written as ∇V
2. The divergence of a vector $\mathbf{A}$, written as $\nabla \cdot \mathbf{A}$
3. The curl of a vector $\mathbf{A}$, written as $\nabla \times \mathbf{A}$
4. The Laplacian of a scalar V, written as $\nabla^2 V$

$$\nabla = \mathbf{a}_\rho \frac{\partial}{\partial \rho} + \mathbf{a}_\phi \frac{1}{\rho} \frac{\partial}{\partial \phi} + \mathbf{a}_z \frac{\partial}{\partial z}$$

$$\nabla = \mathbf{a}_r \frac{\partial}{\partial r} + \mathbf{a}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + \mathbf{a}_\phi \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi}$$

GRADIENT OF A SCALAR

The **gradient** of a scalar field V is a vector that represents both the magnitude and the direction of the maximum space rate of increase of V.



A mathematical expression for the gradient can be obtained by evaluating the difference in the field dV between points P_1 and P_2 of Figure where V_1, V_2, and V_3 are contours on which V is constant. From calculus,

$$dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$
$$= \left(\frac{\partial V}{\partial x} \mathbf{a}_x + \frac{\partial V}{\partial y} \mathbf{a}_y + \frac{\partial V}{\partial z} \mathbf{a}_z \right) \cdot (dx\, \mathbf{a}_x + dy\, \mathbf{a}_y + dz\, \mathbf{a}_z)$$

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For convenience, let

$$\mathbf{G} = \frac{\partial V}{\partial x} \mathbf{a}_x + \frac{\partial V}{\partial y} \mathbf{a}_y + \frac{\partial V}{\partial z} \mathbf{a}_z$$

Then

$$dV = \mathbf{G} \cdot d\mathbf{l} = G \cos \theta \, dl$$

or

$$\frac{dV}{dl} = G \cos \theta$$

$$\text{grad } V = \nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r \sin \theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi$$